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RELATION BETWEEN FLUID VISCOSITY AND COMPRESSIBILITY

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UDC 532.133

Values of the shear viscosity are computed with an error not exceeding the error of experiment by means of formulas deduced on the basis of the theory of a free volume.

Hydraulic systems with working fluid pressures of 30-120 MPa [1] are being used more and more in machine construction. It is known that the shear viscosity and density (compressiblity) of the working fluid depend on the temperature and pressure and exert significant influence on the characteristics and operability of hydraulic system mechanisms as they change, especially when using automatic control instruments [2].

The pressure dependences of the shear viscosity and density of the fluids are determined experimentally on unique specialized instruments and apparatus [3], which require a large expenditure of facilities and time. If an attempt is made to establish a connection between the shear viscosity and density (compressibility) of the fluid, the expenditure in laboratory investigations could be reduced considerably.

Bachinskii [4] proposed the first empirical dependence between the shear viscosity and specific volume (density)

$$\eta = \frac{C}{V - \omega} . \tag{1}$$

Experimental confirmation of the formula yielded satisfactory agreement for a large number of fluids. However, for fluids associated by hydrogen bonds (alcohols, acids, water), some hydrogen halides, and mercury, the formula (1) turned out to be unacceptable. It yields such false results at high pressures. Nevertheless, investigations tracing the connection between the shear viscosity and density (compressibility) of a fluid continued. Thus, the authors of [5] found a relationship between these quantities for individual paraffins, the dependence $\eta = f(\rho)$ was derived in [6] for polysiloxanes, and an attempt was made in [7, 8] to establish such a relation for commercial mineral oils. We try to establish these relationships in an example of two working fluids of a hydraulic system that differ substantially in their chemical nature.

The uninflammable fluid PGV (on a water—glycerine base) is being more and more extensively used at this time in hydraulic systems with elevated fire-safety requirements instead of the shaft oil AU. Properties of the fluids PGV and shaft oil AU are compared in [9].

The pressure dependences of the viscosity of the fluid PGV and the shaft oil AU were determined on a viscosimeter with a rolling ball with a ±2-8% error [10], and the density (compressibility) by a hydrostatic method with ±0.2% error [9, 11].

An equation of state of the fluid (Tait equation) [12] was obtained from the experimental dependence of the compressibility

TABLE 1. Dependence of the Volume of "Incompressible" Molecules on the Pressure and Temperature

P, MPa	$K \cdot 10^3$, m³/kg								
	PGV fluid at T, °K						shaft oil AV at T, °K		
	240,3	273,35	293,65	323,35	353,45	373	293	325,85	363,35
0	0,704	0,700	0,622	0,620	0,611	0,606	0,944	0,923	0,942
20	0,706	0,698	0,639	0,632	0,603	0,616	0,944	0,922	0,920
40	0,707	0,694	0,653	0,638	0,629	0,625	0,940	0,930	0,917
60	0,709	0,692	0,659	0,642	0,632	0,630	0,942	0,928	0,918
80	0,710	0,689	0,662	0,646	0,635	0,632	0,941	0,930	0,915
100	0,712	0,687	0,663	0,647	0,636	0,634	0,940	0,928	0,916

$$\frac{V_0 - V}{V_0} = A \ln \left(1 + \frac{P}{B_0 \exp(-\varkappa T)} \right). \tag{2}$$

At the microscopic level viscous fluid flow is displacement of the fluid molecules relative to each other. The magnitude of the shear viscosity is determined by intermolecular interaction forces which grow as the molecules draw together. The fluid volume diminishes under the effect of external pressure, resulting in the molecules drawing together and, therefore, in growth of the viscosity.

The relation between the fluid volume and its shear viscosity can be established on the basis of the simplest representations presented above [13].

Let us represent the fluid volume in the form of a sum of the volumes of "incompressible" molecules K and a free (or intermolecular) volume V_f

$$V = K + V_f. \tag{3}$$

Following [14], we assume that

$$\eta = D \exp \frac{V}{V_f}. \tag{4}$$

Combining (3) and (4), a general form can be obtained for the dependence of the fluid viscosity on the volume

$$\eta = D \exp \left[\frac{V}{V - K} \right]. \tag{5}$$

Using the equation of state $V = V(P, T)$, we convert (5) to the following

$$\ln \frac{\eta(P, T)}{\eta(0, T)} = \frac{V(P, T)}{V(P, T) - K} - \frac{V(0, T)}{V(0, T) - K}. \tag{6}$$

Let us make a quantitative estimate of the applicability of (6) in an example of experimental data on the viscosity and compressibility of PGV and shaft oil AU. To this end, we solve (6) for K. The values obtained for K are presented in Table 1. As is seen from the table, changes in K with pressure and temperature are comparatively slight, which corresponds qualitatively to the assumption of "incompressible" molecules.

Therefore, if we take the following values for the mean values of the volume of the "incompressible" molecules K: $K = 065 \cdot 10^{-3}$ m³/kg for the fluid PGV and $K = 0.93 \cdot 10^{-3}$ m³/kg for shaft oil AU, then (6) can be represented in the form

For the fluid PGV

$$\ln \frac{\eta(P, T)}{\eta(0, T)} = \frac{V(P, T)}{V(P, T) - 0.65 \cdot 10^{-3}} - \frac{V(0, T)}{V(0, T) - 0.65 \cdot 10^{-3}}, \tag{7}$$

For the shaft oil AU

$$\ln \frac{\eta(P, T)}{\eta(0, T)} = \frac{V(P, T)}{V(P, T) - 0.93 \cdot 10^{-3}} - \frac{V(0, T)}{V(0, T) - 0.93 \cdot 10^{-3}}. \tag{8}$$

TABLE 2. Comparison between Experimental Values of the Shear Viscosity η of PGV and Values of the Viscosity Computed by Means of (7) and (10)

P, MPa	240,65K (−32,5 °C)			273,35 K (0,2 °C)			293,65 K (20,5 °C)		
	η_{exp}	η from (7)	η from (10)	η_{exp}	η from (7)	η from (10)	η_{exp}	η from (7)	η from (10)
0	6,58	—	—	0,201	—	—	0,0505	—	—
10	7,04	6,80	7,10	0,214	0,208	0,211	0,0518	0,0522	0,0526
20	7,56	7,02	7,66	0,227	0,215	0,221	0,0536	0,0540	0,0547
40	8,75	7,48	8,91	0,253	0,229	0,244	0,0579	0,0577	0,0591
60	10,2	7,96	10,37	0,280	0,244	0,268	0,0628	0,0615	0,0638
80	12,0	8,47	12,09	0,307	0,260	0,294	0,0679	0,0655	0,0687
100	14,3	9,00	14,1	0,335	0,277	0,322	0,0734	0,0696	0,0740
σ, %	—	36	1,5	—	15	4	—	3	1,7

P, MPa	323,35 K (50,2 °C)			353,45 K (80,3 °C)			373,15 K (100 °C)		
	η_{exp}	η from (7)	η from (10)	η_{exp}	η from (7)	η from (10)	η_{exp}	η from (7)	η from (10)
0	0,0153	—	—	0,00643	—	—	0,00448	—	—
10	0,0157	0,0158	0,0158	0,00659	0,00666	0,00662	0,00459	0,00464	0,00460
20	0,0162	0,0164	0,0163	0,00673	0,00689	0,00681	0,00472	0,00481	0,00473
40	0,01725	0,0175	0,0174	0,00719	0,00737	0,00719	0,00500	0,00514	0,00498
60	0,0184	0,0187	0,0185	0,00762	0,00786	0,00758	0,00530	0,00549	0,00523
80	0,0197	0,0199	0,0197	0,00808	0,00837	0,00798	0,00561	0,00584	0,00548
100	0,02095	0,0212	0,0209	0,00855	0,00890	0,00838	0,00593	0,00621	0,00574
σ, %	—	1	0,6	—	3	1,2	—	5	2

TABLE 3. Comparison between Experimental Values of the Shear Viscosity η of the Shaft Oil AU and Values of the Viscosity Computed by Means of (8) and (11)

P, MPa	292,95 K (19,8 °C)			325,85 K (52,7 °C)			363,35 K (90,2 °C)		
	η_{exp}	η from (8)	η from (11)	η_{exp}	η from (8)	η from (11)	η_{exp}	η from (8)	η from (11)
0	0,0424	—	—	0,00959	—	—	0,00348	—	—
10	0,0538	0,0523	0,0542	0,0114	0,0117	0,0116	0,00422	0,00415	0,00407
20	0,0696	0,0645	0,0695	0,0136	0,0141	0,0140	0,00480	0,00494	0,00474
40	0,116	0,0487	0,115	0,0202	0,0209	0,0205	0,00641	0,00695	0,00638
60	0,198	0,1524	0,195	0,0296	0,0309	0,0300	0,00856	0,00976	0,00855
80	0,336	0,238	0,336	0,0436	0,0460	0,0442	0,0113	0,0137	0,0114
100	0,574	0,378	0,594	0,0634	0,0691	0,0656	0,0150	0,0193	0,0153
σ, %	—	34	1,7	—	5	2,4	—	16	2,1

In substance, (7) and (8) permit the estimation of the viscosity of the fluid PGV and the shaft oil AU from results of measuring the compressibility by using just one constant.

The accuracy of the viscosity estimates can evidently be raised substantially if the empirical dependence $K = K(T)$ is introduced. Then (6) is converted to

$$\ln \frac{\eta(P,\,T)}{\eta(0,\,T)} = \frac{V(P,\,T)}{V(P,\,T) - K(T)} - \frac{V(0,\,T)}{V(0,\,T) - K(T)} . \tag{9}$$

We obtained the dependence $K = K(T)$ by least squares in the form: $K = 4.54 \cdot 10^{-4} + 6.194 \cdot 10^{-2}/T$ for the fluid PGV, and $K = 8.165 \cdot 10^{-4} + 3.644 \cdot 10^{-2}/T$ for the shaft oil AU.

Therefore, the "improved equations" (7) and (8) will have the following forms

For the fluid PGV

$$\ln \frac{\eta(P,\,T)}{\eta(0,\,T)} = \frac{V(P,\,T)}{V(P,\,T) - (4.54 \cdot 10^{-4} + 6.194 \cdot 10^{-2}/T)} - \frac{V(0,\,T)}{V(0,\,T) - (4.54 \cdot 10^{-4} + 6.194 \cdot 10^{-2}/T)} , \tag{10}$$

For the shaft oil AU

$$\ln \frac{\eta(P,\,T)}{\eta(0,\,T)} = \frac{V(P,\,T)}{V(P,\,T) - (8.165 \cdot 10^{-4} + 3.644 \cdot 10^{-2}/T)} - \frac{V(0,\,T)}{V(0,\,T) - (8.165 \cdot 10^{-4} + 3.644 \cdot 10^{-2}/T)} . \tag{11}$$

TABLE 4. Comparison between Experimental Values of the Shear Viscosity η of "Promgidrol" (industrial hydraulic oil) Fluids and Values of the Viscosity Computed from (9)

Promgidrol P 20, $K=0{,}700-0{,}0005t$

P, MPa	233,15K exp	233,15K from(9)	263,15K exp	263,15K from(9)	293,15K exp	293,15K from(9)	323,15K exp	323,15K from(9)
0	26,6	—	0,550	—	0,0750	—	0,0204	—
10	31,6	30,0	0,596	0,601	0,0784	0,0790	0,0217	0,0214
15	34,2	31,2	0,621	0,616	0,0800	0,0818	0,0223	0,0217
25	39,3	35,4	0,670	0,667	0,0836	0,0862	0,0234	0,0229
40	46,8	42,1	0,752	0,753	0,0891	0,0945	0,0250	0,0244
60	58,1	52,9	0,861	0,879	0,0971	0,1059	0,0271	0,0265
80	70,8	67,4	0,964	1,035	0,1050	0,1169	0,0292	0,0289
100	84,7	87,2	1,060	1,206	0,1150	0,1322	0,0314	0,0315
120	101	114,6	1,160	1,414	0,1250	0,1472	0,0337	0,0340
σ, %	9,2		10,3		10,7		1,9	

Promgidrol P 20-M1, $K=0{,}680-0{,}0005t$

P, MPa	263,15K exp	263,15K from(9)	293,15K exp	293,15K from(9)	323,15K exp	323,15K from(9)
0	0,501	—	0,0785	—	0,0232	—
10	0,521	0,522	0,0815	0,0818	0,0241	0,0240
15	0,532	0,529	0,0829	0,0827	0,0246	0,0244
25	0,554	0,551	0,0859	0,0862	0,0255	0,0252
40	0,587	0,583	0,0907	0,0911	0,0268	0,0265
60	0,636	0,630	0,0975	0,0974	0,0287	0,0281
80	0,689	0,679	0,1050	0,1044	0,0306	0,0300
100	0,745	0,729	0,1130	0,1108	0,0326	0,0320
120	0,809	0,786	0,1210	0,1178	0,0347	0,0339
σ, %	1,5		1,3		1,7	

Promgidrol P 20-M1

P, MPa	353,15K exp	353,15K from(9)
0	0,00955	—
10	0,00984	0,00986
15	0,00998	0,01006
25	0,0103	0,01039
40	0,0108	0,01082
60	0,0114	0,01153
80	0,0122	0,01221
100	0,0130	0,01287
120	0,0138	0,01358
σ, %	1,0	

Promgidrol P 20-M2, $K=0{,}675-0{,}00067t$

P, MPa	233,15K exp	233,15K from(9)	263,15K exp	263,15K from(9)	293,15K exp	293,15K from(9)	323,15K exp	323,15K from(9)	353,15K exp	353,15K (9)
0	26,0	—	0,668	—	0,0995	—	0,0273	—	0,0138	—
10	27,9	27,9	0,706	0,705	0,1030	0,1040	0,0283	0,0284	0,0142	0,0142
15	28,9	29,2	0,724	0,724	0,1050	0,1060	0,0288	0,0288	0,0144	0,0145
25	31,3	31,3	0,765	0,763	0,1090	0,1110	0,0299	0,0298	0,0148	0,0149
40	35,0	34,6	0,829	0,825	0,1160	0,1180	0,0316	0,0315	0,0154	0,0156
60	40,8	40,3	0,924	0,915	0,1250	0,1280	0,0340	0,0337	0,0163	0,0165
80	47,6	46,1	1,030	1,012	0,1350	0,1380	0,0366	0,0358	0,0172	0,0174
100	55,4	53,0	1,150	1,120	0,1460	0,1500	0,0394	0,0382	0,0182	0,0184
120	64,7	61,3	1,280	1,240	0,158	0,1620	0,0424	0,0408	0,0193	0,0193
σ, %	3,0		1,8		2,1		2,0		1,0	

Comparison between the experimental and computed results by using (7), (8), (10), and (11) are presented in Tables 2 and 3.

We took the rms deviation σ, defined as follows

$$\sigma = \sqrt{\frac{\sum\limits_{i=1}^{n} \left(\dfrac{\eta_i^{comp} - \eta_i^{exp}}{\eta_i^{exp}}\right)^2}{n-1}}$$

as the criterion for accuracy of the equation.

We verified (9) additionally on three water—glycerine fluids of the "Promgidrol" type, whose experimental values of the shear viscosity and density (compressibility) were obtained in the same temperature and pressure ranges and on the same apparatus [15].

Comparison between the experimental values of the shear viscosity of the fluids P20, P20-M1, and P20-M2 and their computed valued according to (9) is presented in Table 4.

As is seen from the table, the rms deviation σ for the fluids P20-M1 and P20-M2 does not exceed 1-3% and is 9-10% only for the fluid P20 at low temperatures, which is probably explained by the inadequate accuracy of the experimental determination of the shear viscosity at low temperatures.

The results presented above show that equations of the type (9) permit the computation of values of the fluid viscosity by means of experimental compressiblity data, to an error not exceeding the error in the experiment. A simplified equation of the type (6) can be used for approximate estimates.

NOTATION

P, pressure; T, temperature; V(P, T), specific volume as a function of the state parameters (P, T); η(P, T), shear viscosity as a function of the state parameters (P, T); V_f, free volume; K, volume of the "incompressible" molecules; A, B, $\varkappa$, constants in the Tait equation of state; C, ω, D, experimental constants.

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HEAT EXCHANGE IN GAS FURNACES

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UDC 536.3

The article presents a new method of calculating heat exchange on the heating surfaces in furnaces with a real medium but without scatter, with multiple reflections making a considerable contribution to the heat flow.

1. Statement of the Problem. In calculations of heat exchange the single-zone approximation is widely used: the medium is represented by one zone, and its spectrum is described by integral characteristics. Figure 1 shows the model of a furnace with convex heating surface. Together with the lining and the volume of the furnace, we obtain three isothermal zones. The surfaces are taken to be gray. If the role of the multiple reflections is slight, the calculation may be carried out with the formulas of [1] and of earlier editions beginning in 1934. In gas furnaces the effective optical thickness of the volume may be small, and in the spectrum of the medium, with small dust concentration, "windows" with high conductance of radiant fluxes are obtained. When the surfaces have fairly large reflection coefficients, multiple reflections are obtained which make a considerable contribution to the heat flows. The object of the present work is to take multiple reflections of radiant fluxes with a real spectrum of the medium into account. It is assumed that soot and dust particles are suspended in the gas; these particles do not cause substantial energy dissipation.

A correct solution of the problem with a black heating surface was obtained in [2] but the solution with a reflecting surface was written intuitively. The present author did not succeed in writing the correct solution of the problem with a reflecting heating surface either, but he substantiates a simple roundabout way of solving the problem that is acceptable for practical calculations.

It is expedient to assume that the heat losses through the lining and the convective flows of heat to it are equal. Then the temperature of the lining surface is obtained incidentally in the process of solving the problem. In the given problem of external heat exchange, the temperature of the heating surface is taken to be known. The convective flow on the heating surface is calculated separately, and the density of the resulting radiant flux is determined by the formula

$$q_{\text{p}0} = a_{\text{T}} (\theta - \theta_0), \text{ W/m}^2 , \tag{1}$$

or more accurately,

$$q_{\text{p}0} = K A_0 (\varepsilon_1 \theta - a_1 \theta_0), \tag{2}$$

where $\theta_0 \equiv \sigma T_0^4$; $\theta_* \equiv \sigma T_*^4$; $\theta \equiv \theta_\Gamma \equiv \sigma T_\Gamma^4$. The temperature of the medium $T = T_\Gamma$ is the effective temperature. Specialized literature deals with its calculation. The coefficients a_{T} and K depend on the number, the dimensions, optical properties, and the geometry of the bodies taking part in the heat exchange. The problem consists in deriving the formulas of these magnitudes for a model of the furnace shown in Fig. 1.